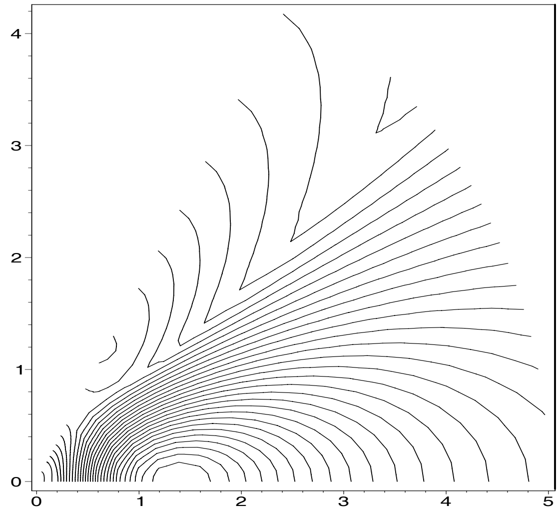
# Criticality in the configuration-mixed Interacting Boson Model : U(5)-QQ mixing

*V. Hellemans[1], P. Van Isacker[2], S. De Baerdemacker[1], and K. Heyde[1]*




*[1]Subatomic and Radiation physics, University of Ghent, Belgium*
*[2]GANIL, Caen, France*

# 1:: Outline

- The interacting boson model and the associated energy surface
- Degenerate critical points and phase diagrams
- Maxwell points
- Spherical-deformed mixing
- Order of the transitions

# 2:: The interacting boson model

IBM approximates nucleon pairs coupled to :

L=0 → s-boson

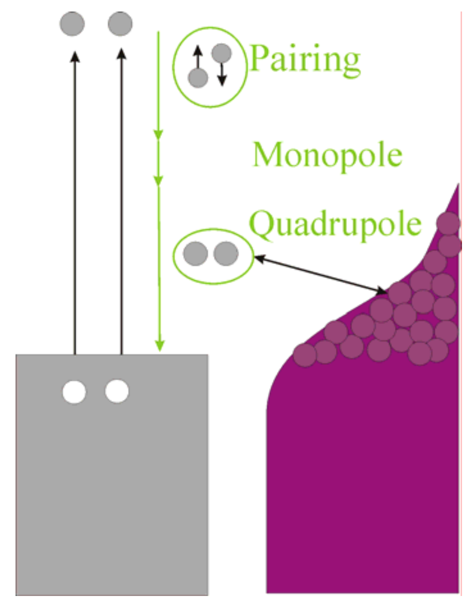
L=2 → d-boson

These bosons constitute an algebra that is analytically solvable in 3 symmetry limits :

U(5) → $\kappa=0$

O(6) → $\varepsilon=0$, $\chi=0$

SU(3) → $\varepsilon=0$, $\chi=\pm 7^{1/2}/2$




The configuration-mixed IBM deals with intruder configurations, p-h excitations out of the closed shell that descend very low in energy!

$$\hat{H} = \hat{H}_{reg} + (\hat{H}_{intr} + \Delta) + \hat{V}_{mix}$$

excitation energy of intruder state, corrected with pairing interaction and monopole correction

takes interaction between configurations into account

# 2:: The interacting boson model

IBM $\xrightarrow[\text{state basis}]{\text{calculate the expectation value in a coherent}}$ Energy surface (ES)

$$\boxed{V_N(\epsilon,\kappa,\chi;\beta,\gamma) = \frac{N\epsilon\beta^2}{1+\beta^2} + \kappa\left[\frac{N(5+(1+\chi^2)\beta^2)}{1+\beta^2} + \frac{N(N-1)}{(1+\beta^2)^2}\left(\frac{2}{7}\chi^2\beta^4 - 4\sqrt{\frac{2}{7}}\chi\beta^3\cos(3\gamma) + 4\beta^2\right)\right]}$$

Configuration mixed IBM $\longrightarrow$ Energy surface

energies are the eigenvalues of

$$\begin{bmatrix} H_N & (V_{mix})_{N,N+2} \\ (V_{mix})_{N+2,N} & H_{N+2}+\Delta \end{bmatrix}$$

energy surface is the lowest eigenvalue of

$$\begin{bmatrix} V_N(\epsilon,\kappa,\chi;\beta,\gamma) & \omega(\beta) \\ \omega(\beta) & V_{N+2}(\epsilon',\kappa',\chi';\beta,\gamma)+\Delta \end{bmatrix}$$

# 3:: Degenerate critical points

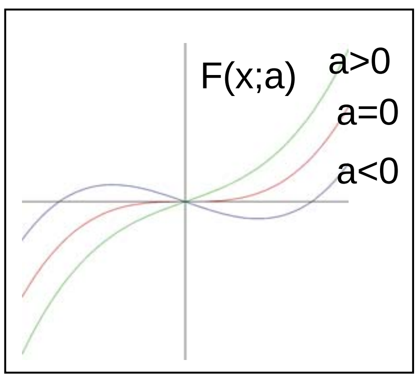
General behaviour of $F(x_1,..,x_n; a_1...a_k)$ ?

→ degenerate critical points mark out the different regions where the qualitative properties of the function remain unchanged

## *One variable*

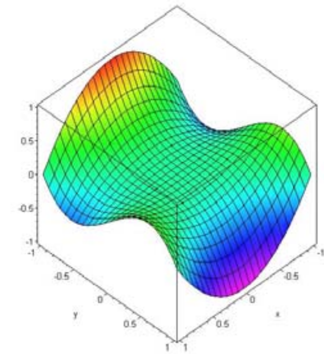
criticality conditions $\frac{\partial F}{\partial x} = \frac{\partial^2 F}{\partial x^2} = 0$

Example




$$F(x;a) = \frac{1}{3}x^3 + ax$$

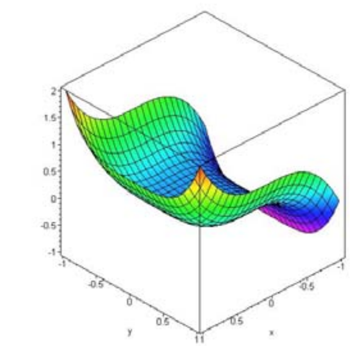
## *Two variables*



criticality conditions

$$\frac{\partial E_-}{\partial \beta} = 0, \frac{\partial E_-}{\partial \gamma} = 0$$

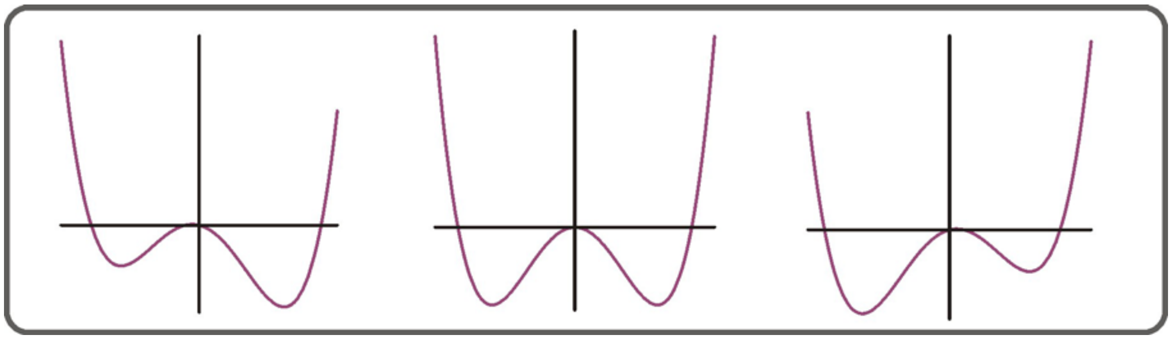
$$det \begin{pmatrix} \frac{\partial^2 E_-}{\partial \beta^2} & \frac{\partial^2 E_-}{\partial \beta\gamma} \\ \frac{\partial^2 E_-}{\partial \gamma\beta} & \frac{\partial^2 E_-}{\partial \gamma^2} \end{pmatrix} = 0$$



Deg. critical points indicate where the qualitative behaviour of the function changes

# 4:: Maxwell points

Maxwell points → of interest in regions where the potential has several minima

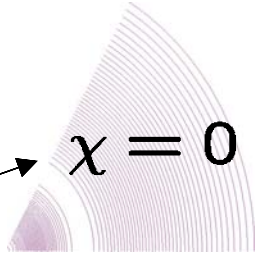


Physically, it is of interest to know which minimum is the global one and where it jumps from one local minimum to another.

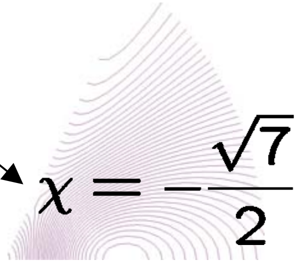
*Maxwell points teach us where this jump occurs*

# 5:: Spherical-deformed mixing

$\epsilon\hat{n}_d$ - $\kappa\hat{Q}(\chi)\cdot\hat{Q}(\chi)$ mixing




U(5)-O(6) mixing




U(5)-SU(3) mixing

$$
\begin{aligned}
E_- := \frac{|\kappa|}{(1+\beta^2)^2} \Big( & ((\epsilon' N - (N+2)(1+\chi^2) - \frac{2(N+2)(N+1)\chi^2}{7} + \Delta')\beta^4 + (\epsilon' N - (N+2)(6+\chi^2) \\
& -4(N+2)(N+1) + 2\Delta')\beta^2 + (\frac{4}{7}(N+2)(N+1)\sqrt{14}\chi)\beta^3\cos(3\gamma) - 5(N+2) + \Delta' \\
& -[((\epsilon' N + (N+2)(1+\chi^2) + \frac{2(N+2)(N+1)\chi^2}{7} - \Delta')\beta^4 + (\epsilon' N + (N+2)(6+\chi^2) \\
& +4(N+2)(N+1) - 2\Delta')\beta^2 - (\frac{4}{7}(N+2)(N+1)\sqrt{14}\chi)\beta^3\cos(3\gamma) \\
& +5(N+2) - \Delta')^2 + \omega'^2(1+\beta^2)^4]^{\frac{1}{2}} \Big)
\end{aligned}
$$

with N the number of bosons, $(\beta,\gamma)$ the collective variables, $\varepsilon'=\varepsilon/|\kappa|$, $\Delta'=\Delta/|\kappa|$, $\omega'=2\omega/|\kappa|$

# 5:: Spherical-deformed mixing

Analytical solution: by expanding around (β,γ)=(0,0)

$$E=t_{00}+\frac{1}{2!}t_{20}\beta^2+\frac{1}{3!}t_{30}\beta^3+\frac{1}{4!}t_{40}\beta^4+\frac{1}{5!}t_{50}\beta^5+\frac{1}{12}t_{32}\beta^3\gamma^2+\dots$$

analytical line in phase diagram

$$\epsilon' = -\frac{(N+2)(4N+\chi^2)}{N}\frac{5(N+2)-\Delta'+\sqrt{(5(N+2)-\Delta')^2+\omega'^2}}{5(N+2)-\Delta'-\sqrt{(5(N+2)-\Delta')^2+\omega'^2}}$$

!! χ is part of a scaling factor : analytical line remains essentialy unchanged when moving from O(6) to SU(3) (or from γ-independant rotor to prolate/oblate rotor)

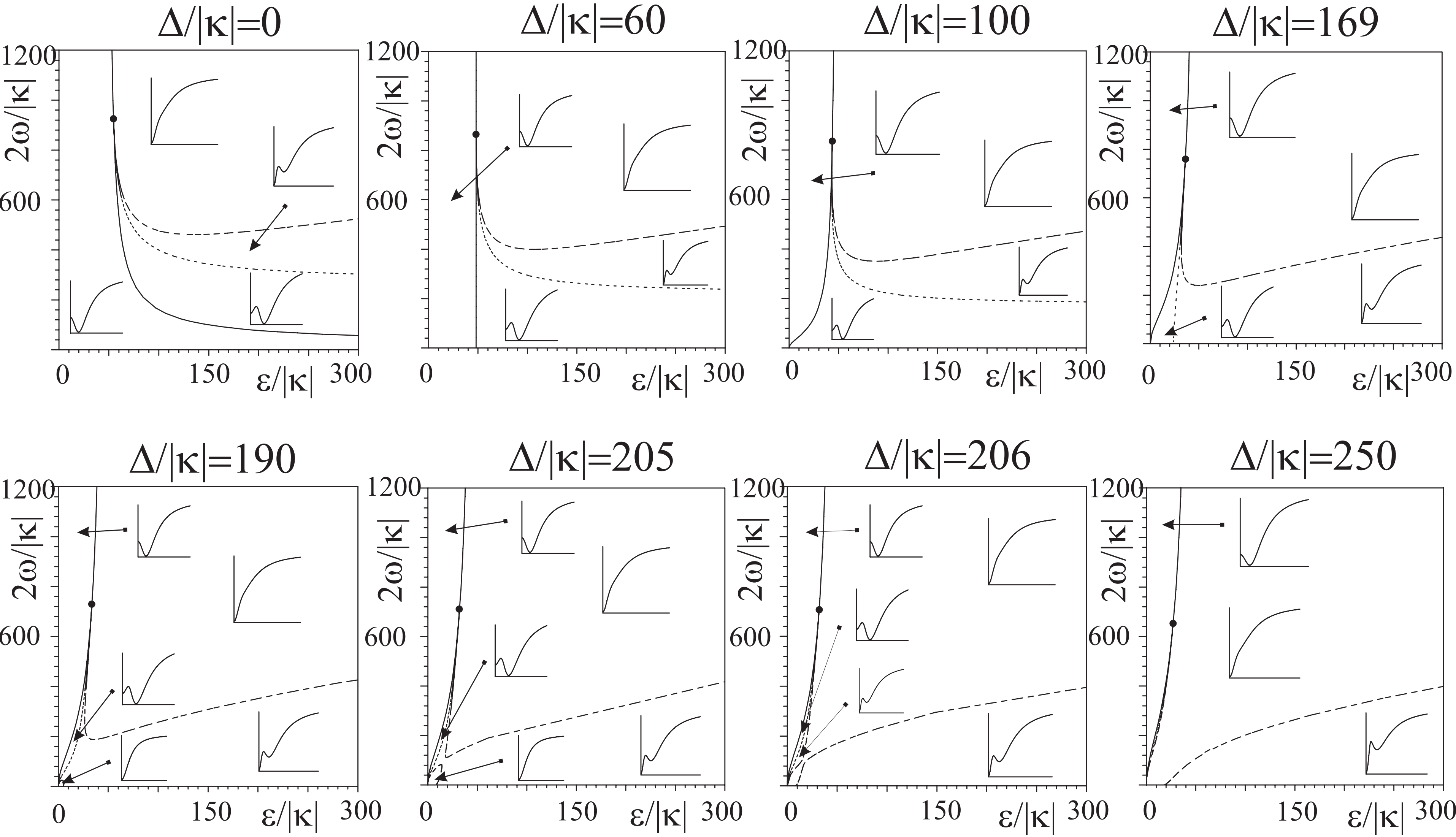
!! on left side of solution : potential has deformed minimum
on right side of solution : potential has spherical minimum

triple points?

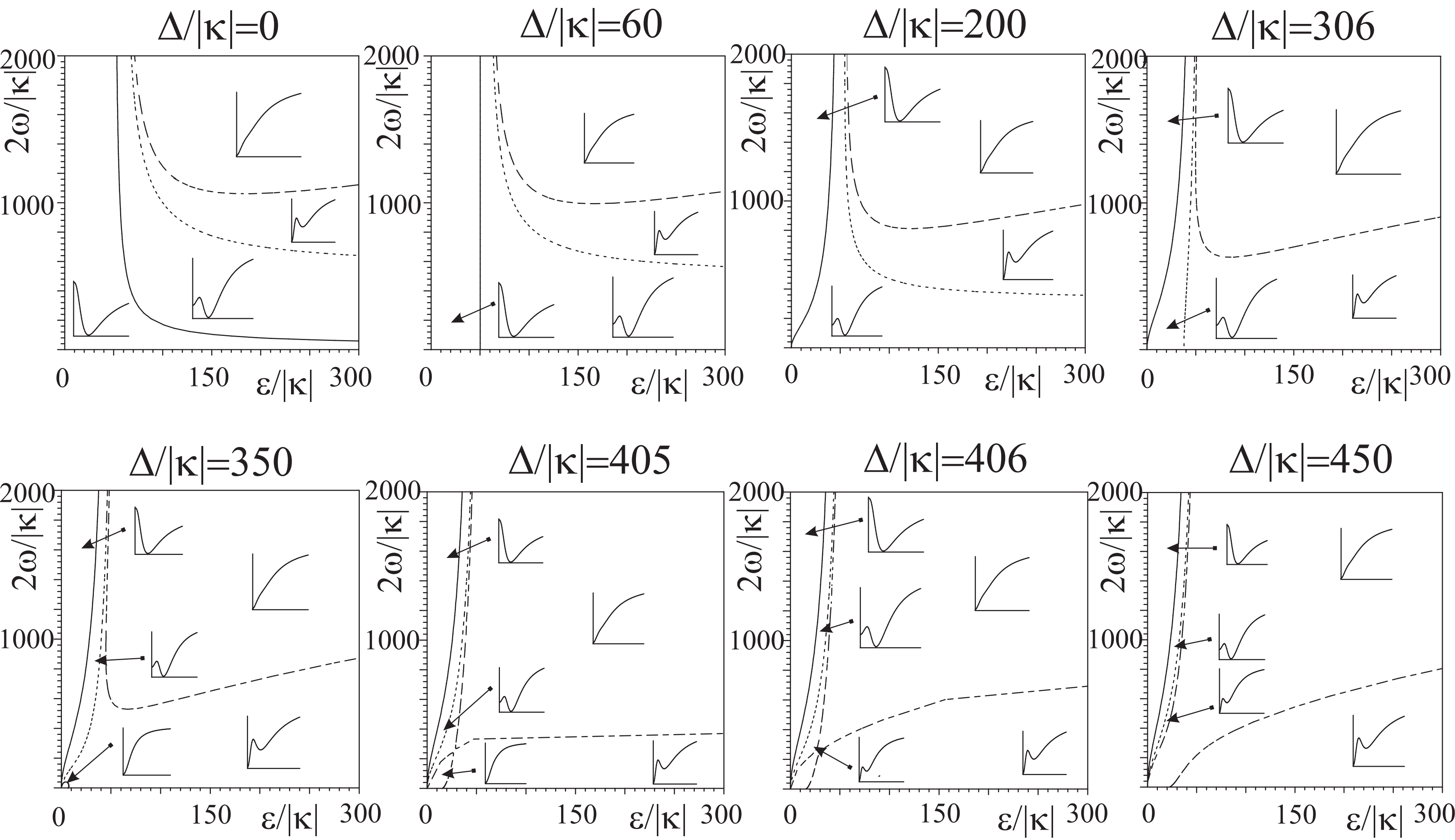
$$t_{30}=\frac{24}{7}(N+2)(N+1)\sqrt{14}\chi\frac{5(N+2)-\Delta'+\sqrt{(5(N+2)-\Delta')^2+\omega'^2}}{\sqrt{(5(N+2)-\Delta')^2+\omega'^2}}$$

if U(5)-O(6) mixing!!

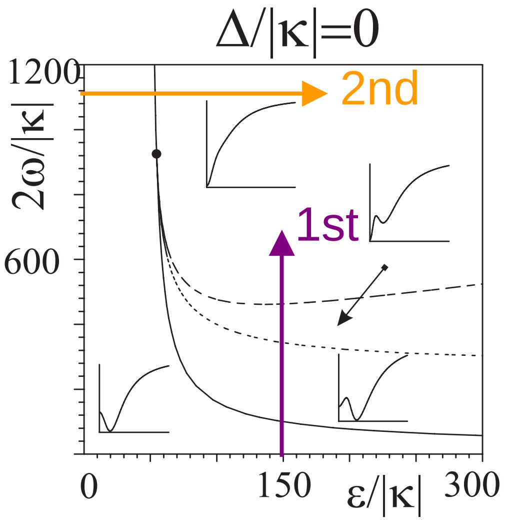
# 5:: Spherical-deformed mixing

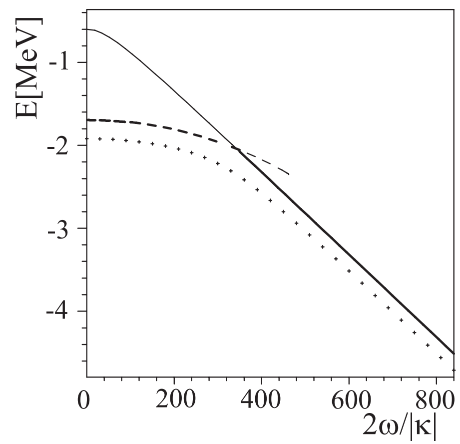


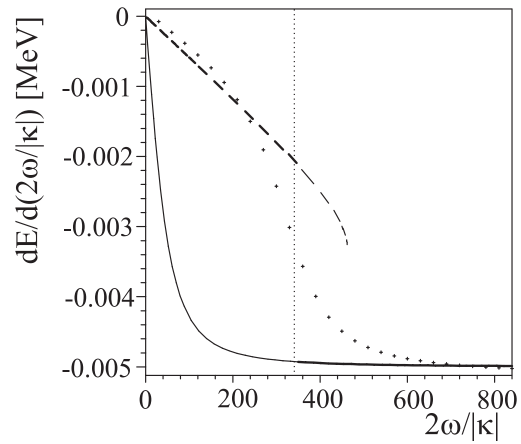
# 5:: Spherical-deformed mixing



# 6:: Order of the transitions



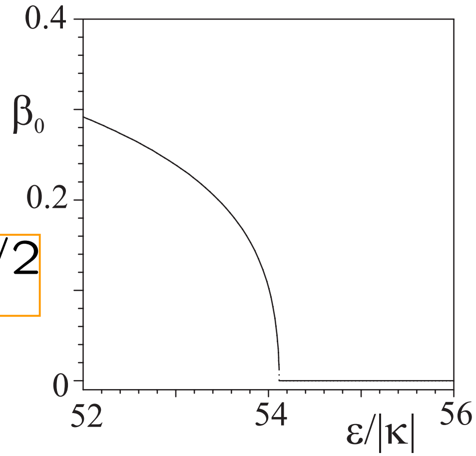



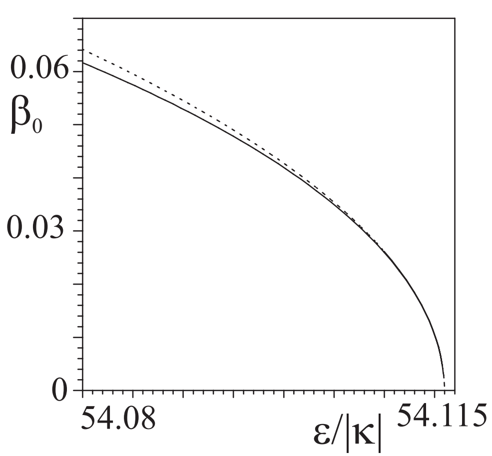



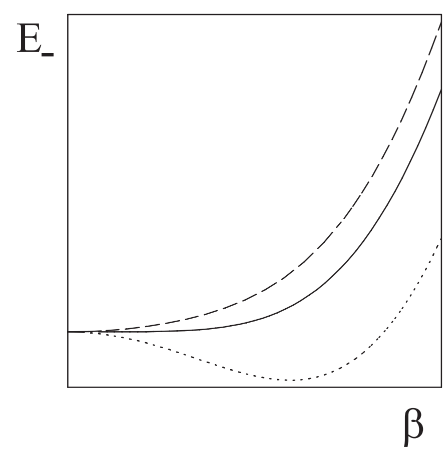

1st order → discontinuity in 1st derivative of energy of the minimum

2nd order → power law

$$\beta_0 \sim |\epsilon'_c - \epsilon'|^{1/2}$$



# 7:: Conclusions and outlook

## Conclusions :

- we calculated the critical points and the maxwell points for U(5)-O(6) and U(5)-SU(3) mixing
- an analytical solution can be found for $(\beta,\gamma)=(0,0)$
  → $\chi$ is part of a scaling factor
  → triple point only in case of U(5)-O(6) mixing : it moves down the analytical solution with increasing $\Delta$
- small region of shape coexistence grows smaller, while the larger region moves away along the $\varepsilon/\kappa$-axis

## To do :

- Study of deformed-deformed mixing? In progress...
- Search for applications where a phase transition occurs
- ...